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# Application of Quadratic Programming Using the Beale Method

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ABSTRACT

Portfolio stock is an alternative for investors in making investment decisions. Portfolio stock can be modeled into a quadratic programming model using mean variance Markowitz which can be solved using the Beale method. This research aims to find the optimal results of portfolio stock. The data used in this research are stock's data on 10 companies that pay out the largest dividends in the IDX High Dividend 20 category for the 2022. There are two selected stock portfolios. The first stock portfolio is a negative covariance stock portfolio with a much different expected return level difference, namely the stock portfolio of PT Astra International Tbk and PT Indo Tambangraya Megah Tbk. The second stock portfolio is a negative covariance stock portfolio with a not much different expected return level difference, namely PT Bank Rakyat Indonesia (Persero) Tbk and PT Mitra Pinasthika Mustika Tbk. Based on Markowitz mean variance, the objective function for the first stock portfolio is $Z = -1{,}305x_1 - 9{,}832x_2 - 0{,}137x_1x_2 + 0{,}681x_1^2 + 2{,}358x_2^2$ with constraints $x_1 + x_2 \leq 100$ and $x_1 + x_2 \geq 0$. After calculating using the Beale method, the expected profit rate is 3.432% and the risk is 0.505% with the optimal proportion for PT Astra International Tbk is 75.063% and the optimal proportion for PT Indo Tambangraya Megah Tbk is 24.937%. In addition, the objective function for the second stock portfolio is $Z = -2{,}426x_1 - 2{,}528x_2 - 0{,}292x_1x_2 + 0{,}366x_1^2 + 0{,}508x_2^2$ with constraints $x_1 + x_2 \leq 100$ and $x_1 + x_2 \geq 0$. With the Beale method, the expected profit rate is 2.471% and the risk is 0.141% with the optimal proportion for PT Bank Rakyat Indonesia (Persero) Tbk is 56.077% and the optimal proportion for PT Mitra Pinasthika Mustika Tbk is 43.923%.



## I. Introduction

Optimization is the process of finding one or more solutions related to the values of one or more objective functions in a problem to get an optimal value [1]. Optimization problems are divided into two, namely optimization problems without constraints and optimization problems with constraints. If the optimization problem with the objective function or constraints is a non-linear function, then the problem is known as non-linear programming.

Quadratic programming is a technique to solve non-linear programming problems with quadratic objective function and linear constraints [2]. One of the methods used to solve quadratic programming is the Beale method. The Beale method is a method that serves to determine the optimal solution for quadratic programming [3]. The Beale method is a modification of the simplex method, where the simplex method is used to solve linear programming. The Beale method uses iterations that will end when the optimal conditions have been met.

Stocks is a form of participation or ownership of a person or business entity in a company [4]. A stock portfolio is a collection of several stocks combined to obtain an optimal return with a certain

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risk [5]. The optimal portfolio can use the Markowitz mean variance model by maximizing return with a certain level of risk or minimizing risk with a certain level of return [6].

IDX High Dividend 20 is index that measures the price performance of 20 stocks that pay cash dividends over the past 3 years and have a high dividend value. The study used stock data of ten companies that paid out the largest dividends in the IDX High Dividend 20 category for the 2022 period. Ten companies that pay the largest dividends in the IDX High Dividend 20 include Adira Dinamika Multi Finance Tbk (ADMF), Astra International Tbk (ASII), Bank Rakyat Indonesia Tbk (BBRI), Bank Mandiri Tbk (BMRI), Hexindo Adiperkasa Tbk (HEXA), Indofood Sukses Makmur Tbk (INDF), Indo Tambangraya Megah Tbk (ITMG), Mitra Pinasthika Mustika Tbk (MPMX), Bukit Asam Tbk (PTBA), and United Tractors Tbk (UNTR). There are two different stock portfolio models formed from the data. Each stock portfolio consists of two companies. To meet the requirements of the minimizing model in the Markowitz mean variance model, the selected stock portfolio is a stock portfolio with a negative covariance. The first stock portfolio is a stock portfolio that has a negative covariance with a much different expected return. While the second stock portfolio is a stock portfolio that has a negative covariance with not much difference expected return. We will see the proportion value of each stock and the optimal results of the two stock portfolio models.

## II. Method

In this study, the company's stock data was obtained from the Indonesia Stock Exchange website at https://www.idx.co.id/id. The purpose of this study is to determine the optimal proportion value of each stock and the optimal results of the two stock portfolio models. The variables used are the adjusted closing value and dividend value of each stock in January 2022 - December 2022. The adjusted closing value is the closing value of the stock in a certain period. Dividends are cash payments made by the company to investor [7]. That data is used to calculate the return. Return is the result obtained from investment. More risk means more return [8]. Returns are divided into individual realized returns, individual expected returns, portfolio realized returns, and portfolio expected returns.

$$R_{it} = \frac{P_{it} - P_{i(t-1)} + (\frac{D}{12})}{P_{i(t-1)}}$$

$$E(R_i) = \frac{\sum_{i=1}^{N} R_{it}}{N}$$

$$R_p = \sum_{i=1}^{N} k_i . R_{it}$$

$$E(R_p) = \sum_{i=1}^{N} (k_i E(R_i))$$

$R_{it}$ : individual realized returns

$P_{it}$ : closing price of the $i$-th stock in period $t$

$P_{i(t-1)}$ : closing price of the $i$-th stock in period $(t-1)$

$D$ : $i$-th stock dividend value

$E(R_i)$ : expected return of $i$-th stock

$N$ : the number of returns that occur in the observation period

$R_p$ : portfolio realized return

$E(R_p)$ : portfolio expected return

$k_i$ : proportion of $i$-th stock

Risk is the amount of deviation between expected return and realized return. The greater the deviation, the greater the level of risk. Because risk is expressed as the amount of deviation from expected results, a measure of dispersion is used, namely variance or standard deviation. Risk is divided into two, namely individual risk and portfolio risk [9].

$$\sigma_i^2 = \frac{\sum_{j=1}^{N}(R_{it} - \left(E(R_i)\right)^2}{N-1}$$

$$\sigma_p^2 = \sum_{i=1}^{N} x_i^2 \sigma_i^2 + \sum_{j=1}^{N} x_j^2 \sigma_j^2 + 2\sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j \sigma_{ij}$$

$$\sigma_{ij} = \frac{\sum_{i,j=1}^{N}\left[R_{it} - \sum_{i=1}^{N} E(R_i)\right][\,R_{jt} - \sum_{j=1}^{N} E(R_j)\ \ ]}{N-1}$$

| | |
|---|---|
| $\sigma_i^2$ | : individual risk |
| $E(R_i)$ | : individual expected return |
| $R_{it}$ | : individual realized returns |
| $N$ | : the number of returns that occur in the observation period |
| $\sigma_p^2$ | : portfolio risk |
| $\sigma_{ij}$ | : risk of $i$-th and $j$-th stock returns |

The results of these calculations are used to form a quadratic programming model based on the Markowitz mean variance model for risk minimization models with a certain level of return, is:

Minimize

$$Z = \sum_{i=1}^{N} -E(R_i)x_i + \sum_{i=1}^{N} x_i^2\sigma_i^2 + \sum_{i=1}^{N}\sum_{j=1}^{N} x_i x_j \sigma_{ij}$$

With constrains:

$$\sum_{i=1}^{N} x_i \leq 1$$

$$x_i \geq 0, i = 1, \dots, N$$

Where:

| | |
|---|---|
| $Z$ | : objective function |
| $E(R_i)$ | : individual expected return |
| $\sigma_i^2$ | : individual risk |
| $\sigma_{ij}$ | : portfolio risk |
| $N$ | : the number of returns that occur in the observation period |

The quadratic programming model is solved using the Beale method to get the optimal proportion value of each stock in the stock portfolio in order to achieve minimum risk. The steps in solving the quadratic programming method of the Beale method for the maximizing case are as follows [10]:

1. Transform the objective function and constraint function of the quadratic programming model into standard form by adding slack variables and/or surplus variables to the inequality constraint function to obtain the equation $Ax = b$ where $b \geq 0$,
2. Selects any m variables as basic variables and (n-m) variables as non-basic variables. It is shown that the basic variables are $X_B = (X_{B_1}.X_{B_2}, \dots, X_{B\_m})$ and the non-basic variables are

$X_{NB} = (X_{NB_1}, X_{NB_2}, \dots, X_{NB_{n-m}})$ where m is the number of constraint functions and n is the number of variables in the objective and constraint functions,

3. Express each basic variables $X_{B_i}$ for each $i = (1,2, \dots, m)$ into the form of non-basic variables $X_{NB_i}$,
4. Express the objective function $Z$ into the form of non-basic variables $X_{NB_i}$,
5. Determine entry variable: Compute the partial derivative of the function $Z$ of non-basic variable $X_{NB}$ where $x_{NB} = 0$
   a. If $\left(\frac{\partial Z}{\partial X_{NB_j}}\right)_{X_{NB}=0} \leq 0$ for each $j = 1,2, \dots, n-m$, then optimal value.
   b. If $\left(\frac{\partial Z}{\partial X_{NB_j}}\right)_{X_{NB}=0} > 0$ for at least one $j$, the choose the largest positive value.

   The selected non-basic variable $X_{NB}$ will become basic variable $X_B$
6. Determine exit variable: Let $X_{NB_j} = X_k$ be entry variable for case (ii) in point 5.
7. Compute min $\left\{\frac{\alpha_{h0}}{|\alpha_{hk}|}, \frac{\gamma_{k0}}{|\gamma_{kk}|}\right\}$ for basic variable $X_h$, where $\alpha_{h0}$ is a constant, $\alpha_{hk}$ is coefficient of $X_j$ in equation of basic variable $X_h$ is converted to the form of non-basic variable, $\gamma_{k0}$ is Constanta dan $\gamma_{kk}$ is cofficient of $X_k$ of $\frac{\partial Z}{\partial X_k}$
   a. If minimum value of $\frac{\alpha_{h0}}{|\alpha_{hk}|}$, then basic variable $X_h$ become exit variable
   b. If minimum value of $\frac{\gamma_{k0}}{|\gamma_{kk}|}$, then a new non-base variable will be formed

$$u_i = \frac{1}{2}\frac{\partial Z}{\partial X_k}$$

## III. Results and Discussion

The formation of a portfolio model in the form of quadratic programming will use mean variance Markowitz by looking for individual realized return values, individual expected returns, variance, and covariance based on adjusted closing stock price data and dividends every month. The stock data used is stock data for January 2022 - December 2022 obtained from the Indonesia Stock Exchange website. The following is the value of individual realized return, individual expected return, variance, and covariance of the stock portfolio.

Table. 1. Individual realized return value

| Period \ Perusahaan | ADMF | ASII | BBRI | BMRI | HEXA |
|---|---|---|---|---|---|
| January 2022 | 0% | 0% | 0% | 0% | 0% |
| February 2022 | 4,333% | 6,383% | 12,169% | 3,431% | 6,189% |
| Marc 2022 | 9,579% | 13,784% | 2,754% | 3,006% | 24,753% |
| April 2022 | -2,282% | 15,581% | 8,977% | 19,091% | 0,003% |
| Mei 2022 | 7,049% | -2,648% | -4,626% | -4,692% | 5,765% |
| June 2022 | -2,114% | -7,162% | -10,050% | -6,411% | -5,509% |
| July 2022 | 2,499% | -4,169% | 5,415% | 4,796% | 5,067% |
| August 2022 | 0,313% | 10,653% | -0,122% | 7,312% | 4,069% |
| September 2022 | 2,461% | -4,676% | 3,795% | 6,837% | 3,956% |
| October 2022 | 4,224% | 0,737% | 3,891% | 12,255% | 2,802% |
| November 2022 | 5,821% | -7,421% | 7,413% | 0,048% | -4,845% |
| December 2022 | 0,006% | -5,397% | -0,508% | -5,415% | -2,880% |
| $E(R_i)$ | 2,657% | 1,305% | 2,426% | 3,355% | 3,328% |
| $\sigma_i^2$ | 0,136% | 0,681% | 0,366% | 0,560% | 0,621% |

Table. 2. Individual realized return value

| Period \ Perusahaan | INDF | ITMG | MPMX | PTBA | UNTR |
|---|---|---|---|---|---|
| January 2022 | 0% | 0% | 0% | 0% | 0% |
| February 2022 | -1,595% | 26,646% | -1,005% | 12,541% | 8,332% |
| Marc 2022 | -3,643% | 9,817% | 17,526% | 6,923% | 3,220% |
| April 2022 | 6,288% | 2,080% | -2,222% | 18,158% | 19,087% |
| Mei 2022 | 5,145% | 40,388% | 1,382% | 20,351% | 7,148% |
| June 2022 | 7,184% | -10,456% | 1,382% | -14,185% | -8,795% |
| July 2022 | -3,204% | 31,264% | -7,834% | 33,993% | 14,250% |
| August 2022 | -8,101% | 0,923% | 5,076% | 0,172% | 5,254% |
| September 2022 | 1,238% | 7,100% | -2,451% | -0,532% | -2,593% |
| October 2022 | 7,438% | 10,357% | 6,122% | -4,859% | -1,151% |
| November 2022 | 0,359% | -5,848% | 13,659% | -1,346% | -1,823% |
| December 2022 | 4,623% | 5,712% | -1,304% | -1,385% | -14,875% |
| $E(R_i)$ | 1,311% | 9,832% | 2,528% | 5,819% | 2,338% |
| $\sigma_i^2$ | 0,243% | 2,358% | 0,508% | 1,752% | 0,882% |

Tabel. 3. Portofolio risk

| Perusahaan | ADMF | ASII | BBRI | BMRI | HEXA | INDF | ITMG | MPMX | PTBA |
|---|---|---|---|---|---|---|---|---|---|
| ASII | 0,041% | | | | | | | | |
| BBRI | 0,452% | 1,928% | | | | | | | |
| BMRI | -0,644% | 3,770% | 0,288% | | | | | | |
| HEXA | 2,04% | 3,733% | 0,787% | 0,536% | | | | | |
| INDF | -0,429% | -1,287% | -0,807% | 0,071% | -1,815% | | | | |
| ITMG | 2,937% | -0,069% | 1,812% | -0,896% | 1,476% | -0,723% | | | |
| MPMX | 1,669% | 1,324% | -0,146% | -0,753% | 2,904% | -0,62% | -4,121% | | |
| PTBA | 1,051% | 2,476% | 3,483% | 2,228% | 3,428% | -1,521% | 2,223% | -4,351% | |
| UNTR | -0,084% | 0,279% | 2,792% | 3,496% | 3,84% | -0,858% | 6,794% | 2,187% | 9,83% |

Then quadratic programming will be formed for both stock portfolios. The first stock portfolio is a portfolio that has a negative covariance with a much different expected return level difference, namely ASII and ITMG, where the covariance value of the two stocks is -0.069% with an expected return from ASII of 1.305% and ITMG's expected return of 9.832%. In addition, there is a risk value of ASII of 0.681% and a risk value of ITMG of 2.358% so that a quadratic programming model will be obtained as follows:

Minimize

$$Z = -1{,}305x_1 - 9{,}832x_2 - 0{,}137x_1x_2 + 0{,}681x_1^2 + 2{,}358x_2^2$$

With constrains:

$$x_1 + x_2 \leq 100$$
$$x_1, x_2 \geq 0$$

The model that has been obtained will be solved by the Beale method. The solution of the Beale method for both stock portfolios is adjusted to the stages of the existing Beale method. Solution of the first stock portfolio is a stock portfolio consisting of ASII and ITMG companies as follows:

Transforming quadratic programming models into standard form

Maximize

$$Z = 1{,}305x_1 + 9{,}832x_2 + 0{,}137x_1x_2 - 0{,}681x_1^2 - 2{,}358x_2^2 \quad 3.1$$

With constraints

$$x_1 + x_2 - s_1 = 100 \quad 3.2$$

$$x_1, x_2 \geq 0 \quad 3.3$$

where $s_1$ is slack variable.

Choose any basic variables and non-basic variables

$$X_B = \{s_1\}$$
$$X_{NB} = \{x_1, x_2\}$$

1st iteration:

Transform $Z$ dan $X_B$ into $X_{NB}$ form

$$Z = 1{,}305x_1 + 9{,}832x_2 + 0{,}137x_1x_2 - 0{,}681x_1^2 - 2{,}358x_2^2 \quad 3.4$$

$$s_1 = x_1 + x_2 - 100 \quad 3.5$$

Determine entry variable by compute partial derivative of function $Z$ of non-basic variable dan defined that $X_{NB} = 0$, so $x_2$ is entry variable.

Determine exit variable using:

$$\min\left\{\frac{\alpha_{10}}{|\alpha_{12}|}, \frac{\gamma_{20}}{|\gamma_{22}|}\right\}$$

so, $X_B = \{x_2\}$, $X_{NB} = \{x_1, s_1\}$

2nd iteration:

Transform 3. 4 dan 3. 5 into $X_{NB}$ form, so

$$x_2 = 100 - x_1 + s_1 \quad 3.6$$

$$Z = -3{,}176x_1^2 - 2{,}358s_1^2 + 4{,}853x_1s_1 + 476{,}861x_1 - 461{,}816s_1 - 22596{,}8 \quad 3.7$$

Determine entry variable by compute partial derivative of function $Z$ of non-basic variabel dan defined that $X_{NB} = 0$, so $x_1$ is entry variable.

Determine exit variable using:

$$\min\left\{\frac{\alpha_{20}}{|\alpha_{21}|}, \frac{\alpha_{10}}{|\alpha_{11}|}\right\}$$

Since entry variable an exit variable same is $x_1$, that formed $X_{NB}$ is $u_1$

$$u_1 = \frac{1}{2}\frac{\partial Z}{\partial x_1} = -3{,}716x_1 + 2{,}427s_1 + 238{,}431 \quad 3.\ 8$$

so, $X_B = \{x_1, x_2\}$, $X_{NB} = \{s_1, u_1\}$

3rd iteration:

Transform 3. 8, 3. 6, dan 3. 7 into $X_{NB}$, so

$$x_1 = 0{,}764s_1 + 75{,}063 - 0{,}315u_1 \quad 3.\ 9$$

$$x_2 = 0{,}236s_1 + 24{,}937 + 0{,}315u_1 \quad 3.\ 10$$

$$Z = 0{,}504s_1^2 - 0{,}315u_1^2 - 0{,}00018s_1u_1 - 97{,}848s_1 - 0{,}0195u_1 - 4699{,}541 \quad 3.\ 11$$

Determine entry variable by compute partial derivative of function $Z$ of non-basic variabel dan defined that $X_{NB} = 0$. Since the value of the partial derivative above is $\leq 0$, the optimal condition has been reached.

So based on the above calculations, the value of $x_1 = 75.063, x_2 = 24.937$ is obtained. With this optimal proportion, the expected return of the stock portfolio is 3.432% and the portfolio risk value is 0.505%.

The second stock portfolio is a portfolio that has a negative covariance with not much different expected return level difference, namely BBRI and MPMX. In the same way as before, the optimal proportion value for BBRI is 56.077% and the optimal proportion value for MPMX is 43.923% with a portfolio expected return value of 2.471% and a portfolio risk of 0.141%.

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