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Stability Analysis of Mathematical Model of *Spread of Covid19 SEIRS* Type with Constant Birth Rate

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ABSTRACT

The Covid19 case dated 11 November 2021 recorded that the human population died from Covid19 (143,595 people) with confirmed cases (4,249,323 cases) and active cases (9,537 cases). Based on these data, it can be concluded that COVID-19 is an acute and deadly disease. In addition to deaths, due to Covid-19, namely the increase in divorce cases, decreased income in the economy and tourism. In this study, the author made a mathematical modeling of Covid19 type $\mathrm{SV_1V_2EIR}$ as an effort to prevent the spread of Covid19. In the modeling there are human populations susceptible to Covid19 (S), human populations have been vaccinated (V_1), human populations have not been vaccinated (V_2), human populations are exposed (E), human populations are infected with Covid19 (I), and human populations recovered from Covid19 (R). The research objectives are 1) to build a mathematical model of Covid19, 2) to determine the fixed point and basic reproduction numbers, and 3) to analyze the stability of the fixed point. This type of research includes applied science research. The research procedure is 1) observing real phenomena, 2) searching literature, 3) determining variables, parameters, and assumptions in mathematical modeling, 4) building a mathematical model of Covid19, 5) analyzing the Covid19 mathematical model in the form of fixed points, basic reproduction numbers, and fixed-point stability. The results of the analysis 1) the mathematical model type $\mathrm{SV_1V_2EIR}$ has a fixed point without disease and an endemic fixed point, 2) a fixed point without disease is stable for the condition $\mathcal{R}_0 < 1$, and the endemic fixed point is stable for the condition $\mathcal{R}_0 > 1$.

I. Introduction

In the end of 2019, the world hit by an outbreak of a deadly disease known as Covid19. Covid19 is a disease caused by the coronavirus and spreads through direct contact with individuals who are already infected with the coronavirus. Spread can occur through touch, coughing, and sneezing. Data on Covid19 cases that occurred in the world dated September 5, 2021, individuals were confirmed positive 218,946,836 people and the death rate reached 4,539,723 people [1]. In Indonesia, corona virus disease is estimated to spread in March 2020. The coronavirus is spreading

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throughout Indonesia. Data on Covid19 cases that occurred in Indonesia, 4,129,020 individuals were confirmed positive with a death rate of 135,861 people [2].

Based on this data, it is necessary to take action as a solution to prevent the spread of Covid19. The Indonesian government has made several efforts in preventing the spread of Covid19 both in medical and non-medical terms. These preventive measures include the implementation of health protocols such as complying with 5M, physical distancing known as physical distancing, and utilizing applied science as a solution to the spread of Covid19 such as mathematical modeling.

According to [3], mathematical modeling was first introduced in 1911 by Ross known as the Ross model. Ross developed a mathematical modeling of the spread of malaria. In 1957, the Ross model refined by MacDonald, known as the Ross-MacDonald model. Nowadays, mathematical modeling continues to develop not only the field of disease, but other areas such as economic, social, agricultural and others. Some of the mathematical modeling that has been developed include Renewable Natural Resources Modeling in Economic Lease Models [4]. Modeling the Spread of Covid19 Infection in Kalimantan [5], An SIR epidemic model for Covid19 spread with fuzzy parameter: the case of Indonesia [6]. Several other studies related to mathematical modeling were conducted by [7]-[8].

In this paper, a mathematical modeling of the spread of Covid19 type SEIRS was developed. The modeling consists of four (4) populations, including Susceptible Population (S), Exposed Population (E), Infected Population (I), and Recovered-population (R). The recovered population is assumed to be a susceptible population caused by a decrease in the body's immune system. The mathematical model is analyzed to determine fixed points of both fixed points without disease and endemic fixed points, their basic reproduction numbers, and fixed-point stability analysis.

II. Literature Review

A. Autonomous Differential Equation of Order-1

System of Autonomous Differential Equations of Order-1:

$$\frac{dx}{dt} = f(x), x \in \mathbb{R}^n \tag{1}$$

with f is a continuous function of real value. Another system of equations i.e a system of inhomogeneous differential equations of order-1 is stated:

$$\frac{dx}{dt} = Ax(t) + b \tag{2}$$

Where b is an ordered non-zero constant vector $n \times 1$.

B. Differential Equation System

The system of differential equations (1) can be written in the form of:

$$\frac{dx}{dt} = Ax \tag{3}$$

where A is an ordered faceted matrix $n \times n$ and the non-zero vector x and $\mathbb{R}^n$ is called the eigen-vector of A. Suppose ξ a scalar that is the eigen value of A, so that it applies:

$$Ax = \xi x \tag{4}$$

or

$$(A - \xi I)x = 0$$
(5)

so that its characteristics equation:

$$det(A - \xi I)x = 0 \tag{6}$$

Vector x is called the eigen-vector corresponding to the eigenvalue ξ .

C. Fixed Point and Fixed-Point Stability

Suppose that a system of differential equations such as (1) is known, then a point is called a fixed point or equilibrium point, if $f(\bar{x}) = 0$. The stability of the fixed point can be seen from its eigenvalues ξ_i, with $i = 1, 2, 3, \dots, n$, by those obtained from $det(A - \xi I) = 0$. The fixed-point stability is determined using the Routh-Hurwitz criterion [9]. The stability of the fixed point has properties [10]:

1. Stable, if
   1) Any real eigenvalue is negative: $\xi_i < 0$ for every i, or
   2) The complex part of eigenvalue $Re(\xi_i) < 0$ for each i.
2. Unstable , if
   1) There is at least one positive real eigenvalue: $\xi_i > 0$
   2) There is at least one complex eigenvalue with $\mathrm{Re}(\xi_i) > 0$.

D. Basic Reproduction Numbers

The basic reproductive number is the expected value of the number of vulnerable populations becoming infected during the time of infection. The basic reproductive number is the dominant eigenvalue of the matrix [11]-[12].

$$G = FV^{-1} \tag{7}$$

with $F = \frac{\partial F_i}{\partial x_j}(x_0)$ a new infection rate matrix and $V = \frac{\partial V_i}{\partial x_j}(x_0)$ an individual displacement rate matrix evaluated at a fixed point.

III. Methods

A. Types of Research

This research is included in the type of applied science research for Covid19**.** The data in this study is data about Covid19 in the form of secondary data derived from references to Covid19 mathematical modeling research. In addition, the data is also sourced from the Website of the Ministry of Health of the Republic of Indonesia.

B. Assumptions in the Model

In making a mathematical model of the spread of Covid19 type SEIRS, there are several assumptions used, including:

1. The birth rate is constant.
2. There is natural mortality for each population.
3. Deaths caused by disease are found only in exposed and infected populations.
4. The recovered population moves to a susceptible population because the body's immune system decreases and recovery is not permanent.

C. Research Procedures

This research was conducted in several steps or stages, including 1) identification of problems regarding Covid19, 2) Literature review, 3) Making mathematical modeling, 4) determining fixed points, 5) Determining basic reproductive numbers, and 6) analysis of fixed-point stability.

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IV. Results

A. SEIRS Type Mathematical Modeling

Mathematical modeling or a comparative diagram of the SEIRS Type of the spread of Covid19 can be seen in Figure 1.

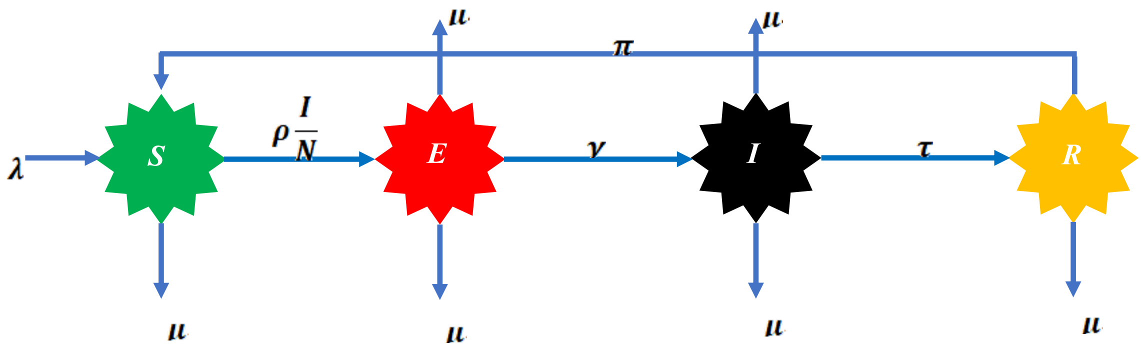

Figure 1. SEIRS Type Mathematical Model

The population of susceptible (S) increases due to the presence of births (λ) and decreases due to natural death (μ_1). In addition, the population is reduced because it moves to an exposed population (E) due to contact between the susceptible - population and the infected population with a chance of infection (ρ). Exposed populations increase due to susceptible populations moving into exposed populations and decreasing due to natural deaths (μ_1), deaths from disease (μ_2), and moving into infected populations at an incubation rate (γ). The infected population (I) increases due to the exposed population moving into an infected population and decreasing due to natural death, death from disease, and moving to a recovered population at the rate of recovery (τ). The recovered population (R) increases due to the infected population moving into the recovered population and decreasing due to natural death. In addition, the recovered - population is reduced because it moves to a susceptible population at a rate at which individuals recover to be revulnerable (π) caused by the body's immune system declining. Based on the compartment diagram in Figure 1, a system of differential equations is obtained, namely:

$$\frac{dS}{dt} = \lambda + \pi R - \left(\mu_1 + \rho \frac{I}{N}\right) S$$
$$\frac{dE}{dt} = \rho \frac{SI}{N} - (\mu_1 + \mu_2 + \gamma) E$$
$$\frac{dI}{dt} = \gamma E - (\mu_1 + \mu_2 + \tau) I$$
$$\frac{dR}{dt} = \tau I - (\pi + \mu_1) R \qquad (8)$$

with $S + E + I + R = N$.

The system of differential equations (8) is simplified by comparing each population to the total population with the aim of facilitating analysis.

$$s = \frac{S}{N}; e = \frac{E}{N}; i = \frac{I}{N}; r = \frac{R}{N} \qquad (9)$$

Systems of equations (8) can be expressed in the form of:

$$\frac{ds}{dt} = \frac{\lambda}{N} + \pi r - (\mu_1 + \rho i) s$$

$$\frac{de}{dt} = \rho si - (\mu_1 + \mu_2 + \gamma)e$$
$$\frac{di}{dt} = \gamma e - (\mu_1 + \mu_2 + \tau)i$$
$$\frac{dr}{dt} = \tau i - (\pi + \mu_1)r \qquad (10)$$

with $s + e + i + r = 1$.

The differential equation system (10) is used to determine fixed points, basic reproductive numbers, and fixed-point stability analysis of the SEIRS type Covid19 spread mathematical model. The units of the parameters used in SEIRS type mathematical models are as shown in Figure 1.

Figure 1. Units of parameters on the model

Parameters	Unit
λ	time^{-1}
μ_1	time^{-1}
μ_2	time^{-1}
ρ	Unsealed
τ	time^{-1}
γ	time^{-1}
π	time^{-1}

B. Fixed Point

The fixed point of a system of differential equations (10) can be determined by means of :

$$\frac{ds}{dt} = 0 \text{ or } \frac{\lambda}{N} + \pi r - (\mu_1 + \rho i)s = 0$$
$$\frac{de}{dt} = 0 \text{ or } \rho si - (\mu_1 + \mu_2 + \gamma)e = 0$$
$$\frac{di}{dt} = 0 \text{ or } \gamma e - (\mu_1 + \mu_2 + \tau)i = 0$$
$$\frac{dr}{dt} = 0 \text{ or } \tau i - (\pi + \mu_1)r = 0 \qquad (11)$$

Based on the analysis carried out on a system of differential equations (11), a disease-free equilibrium and endemic fixed equilibrium point (endemic equilibrium) were obtained. A fixed point without disease satisfies $e = 0,\ i = 0$ and $r = 0$ while an endemic fixed-point load $e \neq 0,\ i \neq 0$ and $r \neq 0$.

$$T_0(s, e, i, r) = \left(\frac{\lambda}{N\mu_1}, 0,0,0\right)$$
$$T_1(s^*, e^*, i^*, r^*)$$

with:

$$s^* = \frac{(\mu_1 + \mu_2 + \gamma)(\mu_1 + \mu_2 + \tau)}{\gamma\rho};$$
$$e^* = \frac{(\mu_1 + \mu_2 + \tau)(\pi + \mu_1)\gamma\rho\lambda - (\mu_1 + \mu_2 + \gamma)(\mu_1 + \mu_2 + \tau)^2(\pi + \mu_1)N\mu_1}{(\mu_1 + \mu_2 + \gamma)(\mu_1 + \mu_2 + \tau)(\pi + \mu_1)\gamma N\rho - \gamma^2 N\rho\pi\tau};$$
$$i^* = \frac{(\pi + \mu_1)\gamma\rho\lambda - (\mu_1 + \mu_2 + \gamma)(\mu_1 + \mu_2 + \tau)(\pi + \mu_1)N\mu_1}{(\mu_1 + \mu_2 + \gamma)(\mu_1 + \mu_2 + \tau)(\pi + \mu_1)N\rho - \gamma N\rho\pi\tau};$$

$$r^* = \frac{\gamma\rho\lambda\tau-(\mu_1+\mu_2+\gamma)(\mu_1+\mu_2+\tau)N\mu_1\tau}{(\mu_1+\mu_2+\gamma)(\mu_1+\mu_2+\tau)(\pi+\mu_1)N\rho-\gamma N\rho\pi\tau} \tag{12}$$

The next stage is carried out an analysis of the stability of fixed points without disease and endemic fixed points.

C. Basic Reproduction Numbers

The basic reproduction number is determined by taking an equation containing only infection with the next generation matrix G approach as in equation (7).

$$\begin{aligned} \frac{de}{dt} &= \rho si - (\mu_1 + \mu_2 + \gamma)e \\ \frac{di}{dt} &= \gamma e - (\mu_1 + \mu_2 + \tau)i \end{aligned} \tag{13}$$

with

$$F_i = \begin{pmatrix} \rho si \\ \gamma e \end{pmatrix} \rightarrow F = \begin{pmatrix} 0 & \frac{\rho\lambda}{N\mu_1} \\ \gamma & 0 \end{pmatrix} \tag{14}$$

and

$$V_i = \begin{pmatrix} (\mu_1 + \mu_2 + \gamma)e \\ (\mu_1 + \mu_2 + \tau)i \end{pmatrix} \rightarrow V = \begin{pmatrix} \mu_1 + \mu_2 + \gamma & 0 \\ 0 & \mu_1 + \mu_2 + \tau \end{pmatrix} \tag{15}$$

$$V^{-1} = \begin{pmatrix} \frac{1}{\mu_1+\mu_2+\gamma} & 0 \\ 0 & \frac{1}{\mu_1+\mu_2+\tau} \end{pmatrix} \tag{16}$$

so that:

$$G = FV^{-1} = \begin{pmatrix} 0 & \dfrac{\rho\lambda}{N\mu_1(\mu_1 + \mu_2 + \tau)} \\ \dfrac{\gamma}{\mu_1 + \mu_2 + \gamma} & 0 \end{pmatrix}$$

Based on the analysis carried out obtained the dominant eigenvalues of the matrix G, namely:

$$\mathcal{R}_0 = \frac{\sqrt{\rho\gamma\lambda}}{\sqrt{N\mu_1(\mu_1+\mu_2+\tau)(\mu_1+\mu_2+\gamma)}} \tag{17}$$

D. Fixed-Point Stability Analysis

Suppose that the system of differential equations (10) is written in the form,

$$\begin{aligned} f_1(s,e,i,r) &= \frac{\lambda}{N} + \pi r - (\mu_1 + \rho i)s \\ f_2(s,e,i,r) &= \rho si - (\mu_1 + \mu_2 + \gamma)e \\ f_3(s,e,i,r) &= \gamma e - (\mu_1 + \mu_2 + \tau)i \\ f_4(s,e,i,r) &= \tau i - (\pi + \mu_1)r \end{aligned} \tag{18}$$

To determine the stability around a fixed point without disease, T_0 first a linearization of the equation (21) is carried out, by means of

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$$J = \begin{pmatrix} \frac{\partial f_1}{\partial s} & \frac{\partial f_1}{\partial e} & \frac{\partial f_1}{\partial i} & \frac{\partial f_1}{\partial r} \\ \frac{\partial f_2}{\partial s} & \frac{\partial f_2}{\partial e} & \frac{\partial f_2}{\partial i} & \frac{\partial f_2}{\partial r} \\ \frac{\partial f_3}{\partial s} & \frac{\partial f_3}{\partial e} & \frac{\partial f_3}{\partial i} & \frac{\partial f_3}{\partial r} \\ \frac{\partial f_4}{\partial s} & \frac{\partial f_4}{\partial e} & \frac{\partial f_4}{\partial i} & \frac{\partial f_4}{\partial r} \end{pmatrix} \tag{19}$$

so that the *Jacobi* matrix is obtained,

$$J = \begin{pmatrix} -(\mu_1 + \rho i) & 0 & -\rho s & \pi \\ \rho i & -(\mu_1 + \mu_2 + \gamma) & \rho s & 0 \\ 0 & \gamma & -(\mu_1 + \mu_2 + \tau) & 0 \\ 0 & 0 & \tau & -(\pi + \mu_1) \end{pmatrix} \tag{20}$$

Fixed points are substituted into the $T_0(s,e,i,r) = \left(\frac{\lambda}{N\mu_1}, 0,0,0\right)$ *Jacobi* matrix (20), obtained matrix :

$$J_{T_0} = \begin{pmatrix} -\mu_1 & 0 & -\frac{\rho\lambda}{N\mu_1} & \pi \\ 0 & -(\mu_1 + \mu_2 + \gamma) & \frac{\rho\lambda}{N\mu_1} & 0 \\ 0 & \gamma & -(\mu_1 + \mu_2 + \tau) & 0 \\ 0 & 0 & \tau & -(\pi + \mu_1) \end{pmatrix} \tag{21}$$

Further determined the eigenvalues using equation (6),

$$\begin{vmatrix} -\mu_1 - \xi & 0 & -\frac{\rho\lambda}{N\mu_1} & \pi \\ 0 & -(\mu_1 + \mu_2 + \gamma) - \xi & \frac{\rho\lambda}{N\mu_1} & 0 \\ 0 & \gamma & -(\mu_1 + \mu_2 + \tau) - \xi & 0 \\ 0 & 0 & \tau & -(\pi + \mu_1) - \xi \end{vmatrix} = 0 \tag{22}$$

The characteristic equation of the matrix equation (22) is:

$$b_0\xi^4 + b_1\xi^3 + b_2\xi^2 + b_3\xi + b_4 = 0 \tag{23}$$

with:

$$b_0 = 1$$
$$b_1 = \pi + 2\mu_1 - \frac{\rho\lambda}{N\mu_1} + (\mu_1 + \mu_2 + \gamma)$$
$$b_2 = \mu_1(\pi + \mu_1) + (\mu_1 + \mu_2 + \gamma)(\mu_1 + \mu_2 + \tau) - \frac{\rho\lambda\gamma}{N\mu_1}$$
$$+(-\pi - 2\mu_1)\left(\frac{\rho\lambda}{N\mu_1} - (\mu_1 + \mu_2 + \gamma)\right)$$
$$b_3 = (\pi + 2\mu_1)\left((\mu_1 + \mu_2 + \gamma)(\mu_1 + \mu_2 + \tau) - \frac{\rho\lambda\gamma}{N\mu_1}\right)$$

$$-(\mu_1(\pi+\mu_1))\left(\frac{\rho\lambda}{N\mu_1}-(\mu_1+\mu_2+\gamma)\right)$$

$$b_4=(\mu_1(\pi+\mu_1))\left((\mu_1+\mu_2+\gamma)(\mu_1+\mu_2+\tau)-\frac{\rho\lambda\gamma}{N\mu_1}\right) \tag{24}$$

Based on the *Routh-Hurwitz* criteria for the characteristic equation of 4, the stability condition is met if:

$$b_1>0, b_3>0, b_4>0 \text{ dan } b_1b_2b_3>b_3^2+b_1^2b_4 \tag{25}$$

A system of differential equations (10) is stable for a fixed point without disease if:

1. $\pi+2\mu_1>\frac{\rho\lambda}{N\mu_1}+(\mu_1+\mu_2+\gamma)$
2. $(\pi+2\mu_1)\left((\mu_1+\mu_2+\gamma)(\mu_1+\mu_2+\tau)-\frac{\rho\lambda\gamma}{N\mu_1}\right)>(\mu_1(\pi+\mu_1))\left(\frac{\rho\lambda}{N\mu_1}-(\mu_1+\mu_2+\gamma)\right)$
3. $(\mu_1+\mu_2+\gamma)(\mu_1+\mu_2+\tau)>\frac{\rho\lambda\gamma}{N\mu_1}$
4. $b_1b_2b_3>b_3^2+b_1^2b_4$

where is the value b_1, b_2, b_3 and b_4 as in the system of equations (26).

Endemic fixed-point stability analysis using a system of differential equations (18). In the same way a linearization of the system of differential equations (18) is performed, so that the Jacobi matrix is obtained and the endemic fixed point is substituted into the *Jacobi* matrix (20), then it is obtained:

$$J_{T_1}=\begin{pmatrix}-(\mu_1+\rho i^*) & 0 & -\rho s^* & \pi\\ \rho i^* & -(\mu_1+\mu_2+\gamma) & \rho s^* & 0\\ 0 & \gamma & -(\mu_1+\mu_2+\tau) & 0\\ 0 & 0 & \tau & -(\pi+\mu_1)\end{pmatrix} \tag{27}$$

Next, the eigenvalue is determined by means of equation (6) around the endemic fixed point (T_1)

$$\begin{vmatrix}-(\mu_1+\rho i^*)-\xi & 0 & -\rho s^* & \pi\\ \rho i^* & -(\mu_1+\mu_2+\gamma)-\xi & \rho s^* & 0\\ 0 & \gamma & -(\mu_1+\mu_2+\tau)-\xi & 0\\ 0 & 0 & \tau & -(\pi+\mu_1)-\xi\end{vmatrix}=0 \tag{28}$$

The characteristic equation of equation (28) is:

$$b_0\xi^4+b_1\xi^3+b_2\xi^2+b_3\xi+b_4=0 \tag{29}$$

with:

$$b_0=1$$

$$b_1=4\mu_1+2\mu_2+\gamma+\tau+\pi+\rho i^*$$

$$b_2=(\mu_1+\mu_2+\gamma)(\mu_1+\mu_2+\tau)-\rho s^*\gamma+(-2\mu_1-\rho i^*-\pi)(-2\mu_1-2\mu_2-\gamma-\tau)$$
$$+(\mu_1+\rho i^*)(\pi+\mu_1)$$

$$b_3=(2\mu_1+\rho i^*+\pi)\left((\mu_1+\mu_2+\gamma)(\mu_1+\mu_2+\tau)-\rho s^*\gamma\right)$$
$$+(2\mu_1+2\mu_2+\gamma+\tau)(\mu_1+\rho i^*)(\pi+\mu_1)+\rho^2s^*i^*\gamma$$

$$b_4 = \big((\mu_1 + \rho i^*)(\pi + \mu_1)\big)\big((\mu_1 + \mu_2 + \gamma)(\mu_1 + \mu_2 + \tau) - \rho s^* \gamma\big) \\ -\rho\big(i^* \pi \gamma \tau - \rho s^* i^* \gamma (\pi + \mu_1)\big) \tag{30}$$

According to the *Routh-Hurwitz* criterion for the characteristic equation of 4, the stability condition is met if:

$$b_1 > 0, b_3 > 0, b_4 > 0 \text{ dan } b_1 b_2 b_3 > b_3^2 + b_1^2 b_4 \tag{31}$$

Therefore, a system of differential equations (10) is stable for endemic fixed points if:

1. $4\mu_1 + 2\mu_2 + \gamma + \tau + \pi + \rho i^* > 0$
2. $(2\mu_1 + \rho i^* + \pi)\big((\mu_1 + \mu_2 + \gamma)(\mu_1 + \mu_2 + \tau) - \rho s^* \gamma\big) + (2\mu_1 + 2\mu_2 + \gamma + \tau)(\mu_1 + \rho i^*)$ $(\pi + \mu_1) + \rho^2 s^* i^* \gamma > 0$
3. $\big((\mu_1 + \rho i^*)(\pi + \mu_1)\big)\big((\mu_1 + \mu_2 + \gamma)(\mu_1 + \mu_2 + \tau) - \rho s^* \gamma\big) > \rho\big(i^* \pi \gamma \tau - \rho s^* i^* \gamma (\pi + \mu_1)\big)$
4. $b_1 b_2 b_3 > b_3^2 + b_1^2 b_4$ (32)

Where the value $s^* \text{dan } i^*$ of the point element remains endemic and the value and b_1, b_2, b_3 and $b_4$as in equation (32). For the condition $\mathcal{R}_0 < 1$ of the parameter value used $\lambda = 0.125$, $\mu_1 = 0.125$, $\tau = 0.04$ and $\pi = 0.01$ [13]. Parameter value $\mu_2 = 0.017$, $\rho = 0.000024$, and $\gamma = 0.2$ [15]. For the condition $\mathcal{R}_0 > 1$ that the parameter value is still the same, but it is assumed that the value $\rho = 0.3$ and $\gamma = 0.2$. The stability properties of the system of differential equations (10) around fixed points without disease and endemic fixed points are summarized in Table 2.

Table 2. Fixed-point stability properties

Condition	Fixed point without disease T_0	Endemic fixed point T_1
$\mathcal{R}_0 < 1$	Stable	Unstable
$\mathcal{R}_0 > 1$	Unstable	Stable

V. Conclusion

Based on the analysis that has been carried out on the mathematical model of the spread of Covid19 type SEIRS, it is concluded that 1) There are two fixed points in the model, namelythe disease-free equilibrium and the endemic equilibrium, and 2) the disease-free equilibrium fixed point is stable for the condition $\mathcal{R}_0 < 1$, while the fixed point is endemic equilibrium stable to the condition $\mathcal{R}_0 > 1$.

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