

# SEII<sub>T</sub>R Model for Diabetes Mellitus Distribution in Case of Insulin and Care Factors

Nur Fajri<sup>a,1,\*</sup>, Sanusi<sup>b,2</sup>, Asmaidi<sup>c,3</sup>

<sup>a</sup> STIMI Meulaboh, Meulaboh-Tapaktuan Street, Langung, 23681, Indonesia

<sup>b</sup> Abulyatama University, Blangbintang Lama Street, Lampoh Keude, Kuta Baro, 24415, Indonesia

<sup>c</sup> Polytechnic of South Aceh, Merdeka Street, Komplek Reklamasi Pantai, Tapaktuan city, 23751, Indonesia

<sup>1</sup> nurfajri.ifar@gmail.com ; <sup>2</sup> san\_andespul@yahoo.co.id; <sup>3</sup> asmedmat@gmail.com

---

## ARTICLE INFO

## ABSTRACT

Article history:  
Accepted

Keywords:  
Diabetes Mellitus  
Model SEII<sub>T</sub>R  
Basic Reproduction Number  
Insulin

This research has been done to learn diabetes mellitus type SEII<sub>T</sub>R with insulin and care factors. Mathematical model type SEII<sub>T</sub>R is a mathematical model of diabetes in which the human population is divided into five groups: susceptible humans (*Susceptible*)  $S$ , exposed (*Exposed*)  $E$ , infected  $I$  without treatment, infected (*Infected*)  $I_T$  with treatment and recovered (*Recovery*)  $R$ . The SEII<sub>T</sub>R model has two fixed points, namely, a fixed position without disease and an endemic fixed point. By using prime reproduction numbers ( $R_0$ ), it is founding that the fixed point without disease is stable if  $R_0 < 1$  and when  $R_0 > 1$ . Then the fixed point without disease is unstable. The simulation shows the effect of giving insulin to changes in the value of the specified reproduction number. If the effectiveness of  $\beta$  decreases, the necessary reproduction number decreases too. Thus, a reduction in the amount of this parameter will be able to help reduce the rate of diabetes mellitus in the population.

Copyright © 2020 Politeknik Aceh Selatan.  
All rights reserved.

## I. Introduction

Diabetes Mellitus is a chronic metabolic disorder that has a significant impact on health, quality of life, patient's life expectancy, and the health care system. DM is characterizing by a condition where the concentration of glucose in the blood is chronically higher than the standard value due to the body's lack of insulin or ineffective insulin function. This disease is known as a disease due to the modern lifestyle [1]. In addition, Diabetes Mellitus is commonly referred to as the silent killer because this disease can attack all organs of the body and cause various kinds of complaints, including eye complaints, cataracts, heart, kidney sexual impotence, wounds painful to heal, lung infections, blood vessel disorders, strokes and so on [2].

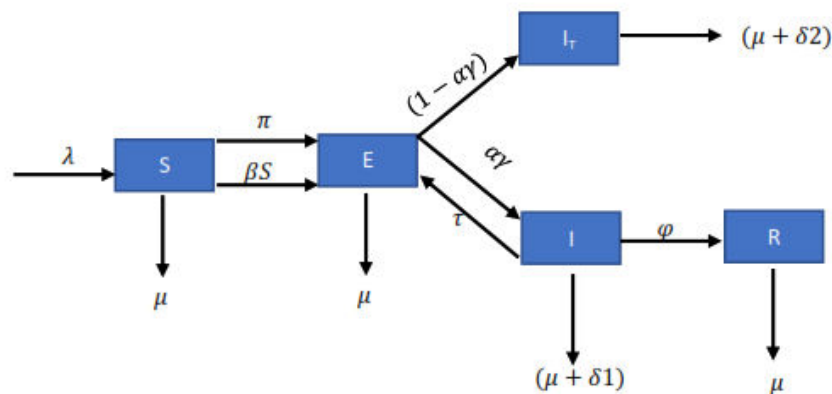
Therefore, action needs to prevent and control diabetes mellitus. Activities that can be taken are by living a healthy lifestyle, administering drugs, administering insulin, and making mathematical modelling of diabetes mellitus.

Several studies on mathematical modelling have been developed by [3], in which the mathematical models of the spread of DHF were developed by involving mosquito populations and mosquito eggs. In 2017, [4] developed a mathematical model of the type of SEIR by using insulin as a form of treatment. According to [5], which formulates the SEII<sub>T</sub> mathematical model into SEII<sub>T</sub>R. Furthermore, analyze the stability of fixed points and simulate the SEII<sub>T</sub>R model. Next [6], developed a mathematical model of diabetes mellitus. In this study, the developed model is a mathematical model of diabetes mellitus type SEIIT by considering treatment and genetic factors.



**II. Research Method**

Math model type  $SEIITR$  is a mathematical model of diabetes in which the human population is divided into five groups, namely the susceptible (S), exposed E, infected (I) without treatment, infected ( $I_T$ ) that receives treatment, and the recovery (R). Individuals included in the susceptible subpopulation are those who have not been exposed to diabetes mellitus. Individuals involved in the exposed subpopulation are individuals who have poor living habits, a decrease in the hormone insulin, and an increase in blood glucose, so these individuals will likely be affected and symptoms of diabetes mellitus. Individuals included in subpopulation I, are individuals who have diabetes mellitus but do not receive treatment. Individuals that are included in the  $I_T$  subpopulation are individuals who have diabetes mellitus and receive treatment. Infected individuals will become R individuals due to insulin administration.



. Figure 1. Math Model Type  $SEIITR$

Based on this, a mathematical model of diabetes mellitus type  $SEIITR$  was developed, as shown in Figure 1. Where  $\lambda$  expresses the birth rate of the population,  $\mu$  expresses the natural rate of death of each population,  $\beta SE$  states the frequency of contact of susceptible populations with exposed populations,  $\pi$  the rate of movement of susceptible populations into populations exposed to the influence of genetic factors,  $\alpha\gamma E$  states the rate of change of the expected people to an infected population,  $\tau$  expresses the speed of movement of an infected population into an exposed population due to insulin administration,  $(1 - \alpha\gamma) E$ , the rate of progression of an exposed people to an  $I_T$  population due to treatment,  $\delta 2 I_T$  states the rate of death due to insulin diseases receiving treatment,  $\delta 1 I$  indicates the rate of death due to illness without medication. The N indicates the total population. The rate of movement of infected people into recovery populations due to insulin administration is denoted by mathematical models of the spread of diabetes are expressed by the following equations:

$$\begin{aligned} \frac{dS}{dt} &= \lambda - (\mu + \beta E + \pi)S \\ \frac{dE}{dt} &= (\beta E + \pi)S - (\mu + 1)E + \tau I \\ \frac{dI}{dt} &= \alpha\gamma E - (\tau + \varphi + \mu + \delta 1)I \\ \frac{dI_T}{dt} &= (1 - \alpha\gamma)E - (\mu + \delta 2)I_T \\ \frac{dR}{dt} &= \varphi I - \mu R \end{aligned} \tag{1}$$

**III. Results and Discussion**

**3.1 Equilibrium Determination**

A fixed point in the dynamic system (1) is obtained based on:

$$\frac{dS}{dt} = \frac{dE}{dt} = \frac{dI}{dt} = \frac{dI_T}{dt} = \frac{dR}{dt} = 0 \quad (2)$$

Based on PDS (1) obtained two types of fixed points, namely fixed points without the disease (*disease-free equilibrium*)  $x_{df}$  and the endemic fixed point (*endemic equilibrium*)  $x_{ee}$ . A disease-free equilibrium is a fixed point containing values  $I = 0$  and  $E = 0$ . Endemic equilibrium is fixed points containing amounts  $I \neq 0$  and  $E \neq 0$ .

- a) Disease-free equilibrium  $x_{df}(S, E, I, I_T, R) = (S^*, 0, 0, 0, 0)$ , where  $S^* = \frac{\lambda}{\pi + \mu}$   
 b) Endemic equilibrium  $x_{ee}(S, E, I, I_T, R) = (S^{**}, E^{**}, I^{**}, I_T^{**}, R^{**})$ , where  

$$S^{**} = \frac{\lambda}{\pi + \beta E^{**} + \mu}, E^{**} = \frac{(\beta E^{**} \pi) S^{**} + \lambda I^{**}}{\mu + 1}, I^{**} = \frac{\alpha \gamma E^{**}}{\lambda + \varphi + \mu + \delta_1}$$

$$I_T^{**} = \frac{(1 - \alpha \gamma) E^{**}}{\mu + \delta_2}, \text{ dan } R^{**} = \frac{\varphi I^{**}}{\mu}.$$

### 3.2 Analysis of disease-free equilibrium stability ( $x_{df}$ )

To examine the stability of disease-free equilibrium  $x_{df}$  can be done by evaluating the Jacobian matrix.

$$J = \begin{pmatrix} \frac{\partial f_1}{\partial S} & \frac{\partial f_1}{\partial E} & \frac{\partial f_1}{\partial I} & \frac{\partial f_1}{\partial I_T} & \frac{\partial f_1}{\partial R} \\ \frac{\partial f_2}{\partial S} & \frac{\partial f_2}{\partial E} & \frac{\partial f_2}{\partial I} & \frac{\partial f_2}{\partial I_T} & \frac{\partial f_2}{\partial R} \\ \frac{\partial f_3}{\partial S} & \frac{\partial f_3}{\partial E} & \frac{\partial f_3}{\partial I} & \frac{\partial f_3}{\partial I_T} & \frac{\partial f_3}{\partial R} \\ \frac{\partial f_4}{\partial S} & \frac{\partial f_4}{\partial E} & \frac{\partial f_4}{\partial I} & \frac{\partial f_4}{\partial I_T} & \frac{\partial f_4}{\partial R} \\ \frac{\partial f_5}{\partial S} & \frac{\partial f_5}{\partial E} & \frac{\partial f_5}{\partial I} & \frac{\partial f_5}{\partial I_T} & \frac{\partial f_5}{\partial R} \end{pmatrix} = \begin{pmatrix} J_{11} & J_{12} & 0 & 0 & 0 \\ J_{21} & J_{22} & J_{23} & 0 & 0 \\ 0 & J_{32} & J_{33} & 0 & 0 \\ 0 & J_{42} & 0 & J_{44} & 0 \\ 0 & 0 & J_{53} & 0 & J_{55} \end{pmatrix} \quad (3)$$

Where

$$J_{11} = -(\mu + \pi), J_{12} = -\beta \left( \frac{\lambda}{\pi + \mu} \right), J_{22} = \beta \left( \frac{\lambda}{\pi + \mu} \right), J_{23} = \tau, J_{32} = \alpha \gamma, J_{21} = \pi, \\ J_{33} = -(\tau + \varphi + \mu + \delta_1), J_{42} = 1 - \alpha \gamma, J_{44} = -(\mu + \delta_2), J_{53} = \varphi, \text{ and } J_{55} = -\mu.$$

The eigenvalue of the Jacobian matrix can be determined by solving determinant  $(J_{x_{df}} - \psi I) = 0$  or can be stated as

$$(J_{44} - \psi)(J_{33} - \psi)(J_{55} - \psi)((J_{11} - \psi)(J_{22} - \psi) - J_{23}J_{32}) - J_{21}J_{12} = 0 \quad (4)$$

According to the matrix  $J_{x_{df}} - \psi I$ , the following five eigenvalues are obtained:

$$\psi_1 = J_{44} = -\mu - \delta_2$$

$$\psi_2 = J_{33} = -\tau - \varphi - \mu - \delta_1$$

$$\psi_3 = J_{55} = -\mu$$

$$\psi_4 = \frac{a_3 + \sqrt{(a_3)^2 - 4a_4}}{2}$$

$$\psi_5 = \frac{a_3 - \sqrt{(a_3)^2 - 4a_4}}{2}$$

Where

$$a_3 = J_{11} + J_{22} = -\beta S + \beta S - (\mu + 1) = -(\mu + 1)$$

$$a_4 = J_{11}J_{22} - J_{23}J_{32} - J_{21}J_{12} \\ = (-\mu - \pi)(\beta S - \mu - 1) - (\tau)(\alpha \gamma) - (\pi)(\beta S)$$

The system will be stable if all eigenvalues are negative [7]. For  $\psi_1, \psi_2$  and  $\psi_3$  always negative, whereas

$$\psi_4 \text{ negative value if } a_3 < 0 \text{ dan } (a_3)^2 > 4a_4$$

$$\psi_5 \text{ negative value if } a_3 < 0.$$

3.3 Determination of Basic Reproduction Numbers

Basic reproduction numbers, denoted by  $R_0$ , is the expected value of the many vulnerable populations that become infected during the infection period. Determining the basic reproduction numbers is done by *The Next Generation Matrix* approach. The next-generation matrix  $G$  is defined as follows:

$$G = FV^{-1} \tag{5}$$

Where F and V as followed

$$F = \begin{pmatrix} \beta \left( \frac{\lambda}{\pi + \mu} \right) & 0 & 0 \\ \alpha\gamma & 0 & 0 \\ 1 - \alpha\gamma & 0 & 0 \end{pmatrix} \text{ and } V = \begin{pmatrix} \mu + 1 & -\tau & 0 \\ 0 & \tau + \varphi + \mu + \delta_1 & 0 \\ 0 & 0 & \mu + \delta_2 \end{pmatrix} \tag{6}$$

Basic reproduction number is the most significant harmful eigenvalue matrix  $K$  noted as

$$R_0 = \frac{K_1 + K_5}{2} + \sqrt{\frac{(K_1 + K_5)^2}{4} - (K_1 K_5 - K_4 K_2)} \tag{7}$$

Where

$$K_1 = \frac{\beta\lambda}{(\mu + 1)(\pi + \mu)}$$

$$K_2 = \frac{-\tau\beta\lambda}{(\mu + 1)(\pi + \mu)(\tau + \varphi + \mu + \delta_1)}$$

$$K_4 = \frac{\alpha\gamma}{(\mu + 1)}$$

$$K_5 = \frac{-\tau(1 - \alpha\gamma)}{(\mu + 1)(\tau + \varphi + \mu + \delta_1)}$$

The enabling conditions of basic reproduction numbers according to [8] are followed;

- a) If  $R_0 < 1$ , then the number of infected humans will decrease in subsequent infections so that the disease will not spread.
- b) If  $R_0 > 1$ , then the number of infected humans will increase during the next infection period, so the disease will spread.

3.4 Parameter value

Table 1. Parameter values on the model SEI<sub>T</sub>R and the unit

Variable	Information	Parameter	Unit	Source
$\lambda$	Birth-rate of the human population	1	Time <sup>-1</sup>	
$\mu$	Natural death rate	0.00123	Time <sup>-1</sup>	
$\beta$	The rate of contact of susceptible populations with exposed populations	0.0009	Time <sup>-1</sup>	
$\tau$	The rate of movement of infected populations into exposed populations due to insulin administration	0.0024	Time <sup>-1</sup>	[9]
$\alpha\gamma$	The rate of movement of exposed populations into the infected populations	0.004	Time <sup>-1</sup>	
$\pi$	The rate of contact of susceptible populations with populations exposed to genetic influences	0.00005	Time <sup>-1</sup>	
$\delta$	Death-rate due to disease	0.00139	Time <sup>-1</sup>	
$\varphi$	Disease rate of the infected population becomes recovery population due to insulin	0.3	Time <sup>-1</sup>	Assumption

3.5 Simulation

In this part of the simulation, we observe population dynamics under conditions when  $R_0 < 1$ , this simulation is needed to show the effect of insulin treatment and administration in humans.

Table 2.  $R_0$  Value with  $\beta$  diminished

$\beta$	$R_0$ Value
0.0009	-0.000015431
0.0019	-0.0000127
0.0029	-0.000010139

Based on Table 2, the smaller the value of  $\beta$ , the lower the basic reproduction number decreases. This can lead to suppressing the rate of spread of diabetes mellitus in the population.

3.5.1 Simulation of the Susceptible Population Contact Rate with the Exposed Population

In this part of the simulation, the population is observed under conditions when  $R_0 < 1$ .  $R_0$  is a basic reproduction number defined in equation (7). This simulation is needed to show the effect of contact rates of susceptible, exposed, infected, infected treatment, and recovery populations on  $\beta$  values. The values of the parameters taken are presented in Table 1 with initial values  $S = 300$ ,  $E = 395$ ,  $I = 2$ ,  $I_T = 400$ , and  $R = 10$ . The value of the basic reproduction number when  $\beta$  is diminished can be seen in Table 2. Simulations of contact rates of susceptible populations with exposed populations are shown in the following figure:

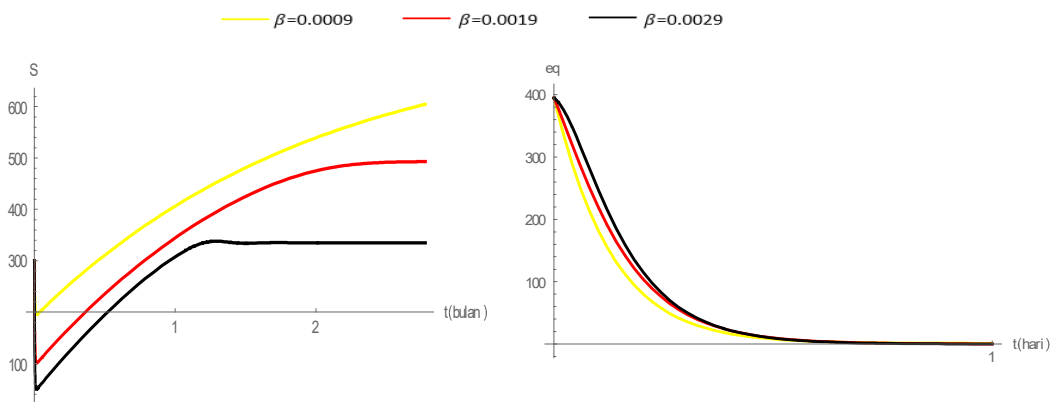


Figure 2 the dynamics of the human population for diminished value  $\beta$

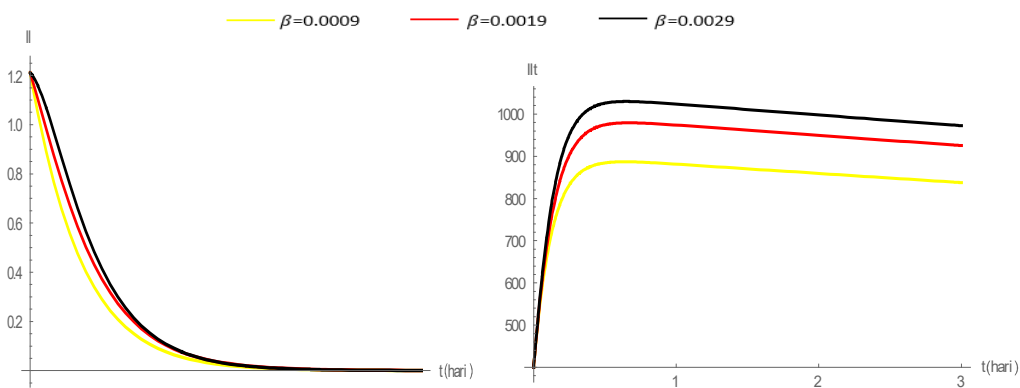


Figure 3 The dynamics of the human population for  $\beta$  value diminished.

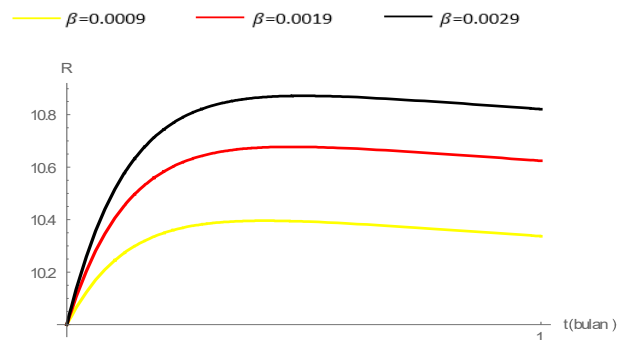


Figure 4 The dynamics of the human population for  $\beta$  value diminished.

In the susceptible class, the simulation shows that the lower the  $\beta$  value, the more humans are in the responsive class. In contrast, in the exposed, infected, infected treatment and recovery class, the lower the  $\beta$  value, the fewer people are in the infected, infected treatment and recovery class. This causes the smaller the value it will help reduce the rate of spread of diabetes mellitus.

#### IV. Conclusion

The details of the main results in this study are as follows:

1. In the model of the spread of diabetes mellitus type  $SEIITR$ , there are two equilibrium, disease-free equilibrium, and endemic equilibrium. If  $R_0 < 1$ , so the equation 3.3 has a stable disease-free equilibrium  $x_{df}$ . If  $R_0 > 1$ , then  $x_{df}$  becomes unstable.
2. The simulation shows the effect of giving insulin to changes in the value of the basic reproductive number ( $R_0$ ). If effectivity  $\beta$  is decreasing, then the basic reproduction number decreases. Thus, a decrease in the amount of this parameter will be able to help reduce the rate of diabetes mellitus in the population.

#### Acknowledgement

Thank you to the Chairperson of the STIMI Management Economics Study Program, Meulaboh. My thanks to my fellow lecturers who have guided and assisted in the publication of my journal. Hopefully, I get this guidance and assistance to improve myself in future journal publications.

#### References

- [1] Subroto, M. Ahkam 2006. *Ramuan Herbal untuk Diabetes Mellitus*. Depok: Penebar Swadaya
- [2] Trisnawati, S. K Setyorogo. S. 2013. *Faktor Resiko Kejadian Diabetes Melitus Tipe II Di Puskesmas Kecamatan Cingkareng Jakarta Barat Tahun 2012*. Jurnal Ilmiah Kesehatan, 5(1) : pp. 6-11
- [3] Asmaidi A, Sianturi P, Nugrahani EH. A SIR Mathematical Model of Dengue Transmission and its Simulation. IAES. 2014; 2(11)
- [4] Asmaidi, Suryanto, ED. 2017. *Pemodelan Matematika Tipe SEIR Diabetes Mellitus*. Seminar Nasional Teknologi Rekayasa - 4 (SNTR-4), ISSN 2407 – 8735, 2 Desember 2017. Politeknik Aceh Selatan
- [5] Lestari HE, Lestari D, Arifah H. 2017. *Analisis Kestabilan Model SEIIT Pada Penyakit Diabetes Mellitus*. Jurnal Matematika
- [6] Asmaidi, Suryanto, ED. 2019. *Mathematics Modeling of Diabetes Mellitus Type SEIIT by Considering Treatment and Genetics Factors*. IOP Conference Series: Materials Science and Engineering, 506(1), April 2019. Politeknik Aceh Selatan
- [7] Tu PNV. 1994. *Dynamical Sistem*. New York: Springer-Verlag

- [8] Van den Driessche P, Watmough J. 2008. Chapter 6: *FutherNothes on the BasicReproduction Number. Mathematical Epidemiology, Lecture Notes, Mathematics*. New York: Springer. 1945: 159-178.
- [9] Asmaidi. 2017. *Mathematical Modeling of SEIR Type to Controlling Diabetes Mellitus Disease Using Insulin*. Inotera. (2)-8-17.